

Axial All-Optical Super Resolved Imaging

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Abstract

A new patented technology that is briefly described in this paper proposes an extended depth of focus element configured as binary phase-affecting, non-diffractive optical element defining a spatially low frequency phase transition that codes the lens aperture. Since this optical element contains low spatial frequencies, it is not sensitive to wavelengths and does not scatter energy towards the outer regions of the field of view. The optical element is a phase element and thus does not cause apodization and its energetic efficiency is very high. Since the optical element does not require digital post processing it is adequate for ophthalmic applications. The technology presented in this paper was developed by Xceed Imaging Ltd.

1. Introduction

Extending the depth of focus of imaging systems is a very important core technology that can be incorporated into various camera aided and ophthalmic applications. Several previous approaches were suggested involving digital post processing [1-4], or aperture apodization by absorptive mask [5-9], or diffraction optical phase elements such as multi focal lenses or spatially dense distributions that suffer from significant divergence of energy into regions that are not the regions of interest [10-12].

The technology described in this paper proposes an extended depth of focus (EDOF) element configured as a phase-affecting, binary optical element defining a spatially low frequency phase transition that codes the lens aperture [13]. This element exhibits several important features: since this optical element contains low spatial frequencies, it is not sensitive to wavelengths and dispersion (as other diffractive optical elements do) and it does not scatter energy towards the outer regions of the field of view. In addition its fabrication is simple and cheap. The optical element is a phase only element and thus it does not cause apodization and its energetic efficiency is very high. The energetic efficiency is high and close to 100% also due to the fact that the element has no spatial high frequencies and thus there is no un-used energy directed to diffraction orders. Since the optical element does not require digital post processing it is adequate for ophthalmic applications.

The optical element is a mask constructed out of transparent areas and binary phase lines (e.g., grid) and/or one or more binary phase circles that modulate the entrance pupil of the imaging lens. Under spatially incoherent illumination the out of focus effect is expressed as a quadratic phase distortion added to the OTF (Optical Transfer Function). The positions of those binary phase transitions are appropriately selected to generate invariance to these quadratic phase distortions. The developed element is also not sensitive to its transversal, as well as longitudinal position, and is thus very suitable to be placed on eye glasses. For obtaining transversal insensitivity low spatial frequency, periodic

replication of the mask contours (the contour or transition regions) is generated.

The position of the binary phase transitions is computed using iterative algorithm in which M positions are examined and eventually those that provide maximal contrast of the OTF under a set of out of focus locations are chosen. The meaning of OTF's contrast optimization is actually having the out of focused OTF bounded as much as possible away from zero.

This technology [13] was tested experimentally and showed operation under severe defocusing conditions (see mathematical definition later on).

Section 2 presents the theoretical background and the mathematical derivation. In section 3 one may see some experimental results. The paper is concluded in section 4.

2. Theoretical Derivation

The OTF of an imaging system can be expressed as an auto-correlation operation between the pupil function of the lens [14]:

$$H(\mu_x, \mu_y; Z_i) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P\left(x + \frac{\lambda Z_i \mu_x}{2}, y + \frac{\lambda Z_i \mu_y}{2}\right) P^*\left(x - \frac{\lambda Z_i \mu_x}{2}, y - \frac{\lambda Z_i \mu_y}{2}\right) dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |P(x, y)|^2 dx dy} \quad (1)$$

While in focus, $P(x, y)$ is the binary circle pupil function, which is "1" within the pupil, and "0" outside. When aberrations are introduced, the generalized pupil function can be described as:

$$P(x, y) = |P(x, y)| \exp[ikW(x, y)] \quad (2)$$

Where $W(x, y)$ is the wave aberration and $k=2\pi/\lambda$ while λ is the optical wavelength. If the aberrations are caused only by defocusing, $W(x, y)$ has the form of:

$$W(x, y) = W_m \frac{(x^2 + y^2)}{b^2} \quad (3)$$

where b is the radius of the aperture P . The coefficient W_m determines the severity of the error. The coefficient W_m is also denoted as:

$$W_m = \frac{\Psi \lambda}{2\pi} \quad (4)$$

where Ψ is a phase factor represents the severity of out of focus:

$$\Psi = \frac{\pi b^2}{\lambda} \left(\frac{1}{Z_i} + \frac{1}{Z_o} - \frac{1}{F} \right) \quad (5)$$

where $2b$ is the diameter of the lens, λ is the wavelength, Z_o is the distance between the imaging lens and the object, Z_i the distance between the imaging lens and the sensor and F is the focal length. When imaging condition is fulfilled:

$$\frac{1}{Z_i} + \frac{1}{Z_o} = \frac{1}{F} \quad (6)$$

and thus the distortion phase Ψ equals zero. For the sake of simplicity we will perform a 1-D analysis. We assume that a phase mask consisting out of a set of phase shifts is attached to

the entrance pupil of a lens and thus it multiplies the generalized pupil function (CTF plane). The OTF which is the auto correlation of the CTF (see Eq. 1) will thus be:

$$T(x) \equiv P(x) \sum_{n=1}^N \exp\left(ia_n \operatorname{rect}\left(\frac{x - n\Delta x}{\Delta x}\right)\right) \quad (7)$$

$$H(\mu; Z_i) = \frac{\int_{-\infty}^{\infty} T\left(x + \frac{\lambda Z_i \mu}{2}\right) T^*\left(x - \frac{\lambda Z_i \mu}{2}\right) dx}{\int_{-\infty}^{\infty} |P(x)|^2 dx}$$

where a_n are binary coefficients equal either to zero or to a certain phase modulation depth: $a_n = (0, \Delta\phi)$ of the phase only element that we design. $\Delta\phi$ is the phase depth of modulation. Δx represents the spatial segments of the element. Since we do not want to create a diffractive optical element, i.e. spatial high frequency periodicity (such that there will be no wavelength dependence) we force $\Delta x \gg \lambda$. The mathematical formulation for the optimization criteria will be as follows: Compute a phase only element that will provide maximum for the minimal value of the OTF within desired spectral region of interest while the OTF is composed out of two terms. The first is the OTF having strong defocusing deformation with parameter of W_m and the second term is the in-focused OTF, i.e.:

$$\max_{a_n, \Delta x} \left\{ \min_{\mu_x} \{H(\mu_x; Z_i, W_m)\} + \min_{\mu_x} \{H(\mu_x; Z_i, 0)\} \right\} \quad (8)$$

Let us now do the approximated derivation: Without any mask on the lens aperture the one-dimensional OTF equals to [14]:

$$H(\mu, W_m) = \frac{\int_{-\infty}^{\infty} P\left(x + \frac{\lambda Z_i \mu}{2}\right) P^*\left(x - \frac{\lambda Z_i \mu}{2}\right) dx}{\int_{-\infty}^{\infty} |P(x)|^2 dx} = \frac{\int_{A(\mu)} \exp\left[\frac{ikW_m 2\lambda Z_i \mu x}{b^2}\right] dx}{2b} \quad (9a)$$

where $A(\mu)$ is the lens aperture size. This can be approximated as:

$$H(\mu, W_m) \approx \frac{\int_{A(0)} \exp\left[\frac{ikW_m 2\lambda Z_i \mu x}{b^2}\right] dx}{2b} = \frac{\int_{-b}^b \exp\left[\frac{4\pi i W_m Z_i \mu x}{b^2}\right] dx}{2b} \quad (9b)$$

$$= \frac{1}{2b} \cdot \int_{-\infty}^{\infty} \operatorname{rect}\left(\frac{x}{2b}\right) \exp\left[\frac{4\pi i W_m Z_i \mu x}{b^2}\right] dx = \frac{\sin\left(\frac{4W_m \pi \mu Z_i}{b}\right)}{2b} \quad (9b)$$

The approximation is true for μ values which are not too large in comparison to $b/\lambda Z_i$ and therefore $A(\mu)$ can be approximated as $A(0)$ or for large W_m values causing the argument of the exponent to oscillate rapidly. Next we assume that the phase element carries only one stripe of non-zero phase. When such an element is attached to the aperture one obtains:

$$H(\mu, W_m) \approx \frac{1}{2b} \int_{-b}^b \exp\left[\frac{4\pi i W_m Z_i \mu x}{b^2}\right] \exp\left[i\Delta\phi \cdot \operatorname{rect}\left(\frac{x + \lambda Z_i \mu / 2}{\Delta x}\right)\right] \cdot \exp\left[-i\Delta\phi \cdot \operatorname{rect}\left(\frac{x - \lambda Z_i \mu / 2}{\Delta x}\right)\right] dx \quad (10)$$

For μ large enough (but not too large in comparison to $b/\lambda Z_i$ otherwise the approximation of Eq. 9 will not be valid) one may approximate:

$$\exp\left[i\Delta\phi \cdot \operatorname{rect}\left(\frac{x + \lambda Z_i \mu / 2}{\Delta x}\right)\right] \exp\left[-i\Delta\phi \cdot \operatorname{rect}\left(\frac{x - \lambda Z_i \mu / 2}{\Delta x}\right)\right] \approx 1 + [\exp(i\Delta\phi) - 1] \operatorname{rect}\left(\frac{x + \lambda Z_i \mu / 2}{\Delta x}\right) + [\exp(-i\Delta\phi) - 1] \operatorname{rect}\left(\frac{x - \lambda Z_i \mu / 2}{\Delta x}\right) \quad (11)$$

Thus the expression for the OTF becomes:

$$H(\mu, W_m) \approx \frac{1}{2b} \int_{-\infty}^{\infty} \operatorname{rect}\left(\frac{x}{2b}\right) \left\{ 1 + [\exp(i\Delta\phi) - 1] \operatorname{rect}\left(\frac{x + \lambda Z_i \mu / 2}{\Delta x}\right) + [\exp(-i\Delta\phi) - 1] \operatorname{rect}\left(\frac{x - \lambda Z_i \mu / 2}{\Delta x}\right) \right\} \exp\left[\frac{4\pi i W_m Z_i \mu x}{b^2}\right] dx \quad (12)$$

which yields the following expression:

$$H(\mu, W_m) \approx \frac{1}{2b} \left\{ \delta(\mu) + [\exp(i\Delta\phi) - 1] \sin c\left(\frac{2\pi W_m Z_i \mu \Delta x}{b^2}\right) \exp\left(\frac{-2\pi i W_m \lambda Z_i^2 \mu^2}{b^2}\right) + [\exp(-i\Delta\phi) - 1] \sin c\left(\frac{2\pi W_m Z_i \mu \Delta x}{b^2}\right) \exp\left(\frac{2\pi i W_m \lambda Z_i^2 \mu^2}{b^2}\right) \right\} \otimes \sin c\left(\frac{4\pi W_m Z_i \mu}{b}\right) \quad (13)$$

where \otimes represents the convolution operation. The last expression equals to:

$$H(\mu, W_m) \approx \frac{\sin c\left(\frac{4\pi W_m Z_i \mu}{b}\right)}{2b} \left\{ \delta(\mu) + 2 \sin c\left(\frac{2\pi W_m Z_i \mu \Delta x}{b^2}\right) \left[\cos\left(\frac{2\pi W_m \lambda Z_i^2 \mu^2}{b^2} - \Delta\phi\right) - \cos\left(\frac{2\pi W_m \lambda Z_i^2 \mu^2}{b^2}\right) \right] \right\} \quad (14)$$

Using the trigonometric relation of:

$$\cos \alpha - \cos \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\beta - \alpha}{2}\right) \quad (15)$$

one obtains:

$$H(\mu, W_m) \approx \frac{\sin c\left(\frac{4\pi W_m Z_i \mu}{b}\right)}{2b} + \frac{4 \sin\left(\frac{\Delta\phi}{2}\right)}{2b} \sin c\left(\frac{4\pi W_m Z_i \mu}{b}\right) \otimes \left\{ \sin c\left(\frac{2\pi W_m Z_i \mu \Delta x}{b^2}\right) \sin\left(\frac{2\pi W_m \lambda Z_i^2 \mu^2}{b^2} - \frac{\Delta\phi}{2}\right) \right\} \quad (16)$$

The first term is the regular term obtained due to getting out of focus and the second term is related to the influence of the binary phase only element. If our phase mask will contain several phase transitions rather than one as was derived in the mathematical analysis, then a summation will appear in front of the second term of Eq. 16. The last expression allows to extract easily both the derivative of H in respect to Δx or $\Delta\phi$ as well as to see the maximum of the minimum of H as required by the optimization criteria of Eq. 8.

Now let us derive the obtained expression in the case that we use the mask but the object is in-focus. In this case the OTF according to Eqs. 1 and 7 will be:

$$H(\mu, 0) = \frac{1}{2b} \int_{-\infty}^{\infty} \operatorname{rect}\left(\frac{x + \lambda Z_i \mu / 2}{\Delta x}\right) \operatorname{rect}\left(\frac{x - \lambda Z_i \mu / 2}{\Delta x}\right) \exp\left[i\Delta\phi \cdot \operatorname{rect}\left(\frac{x + \lambda Z_i \mu / 2}{\Delta x}\right)\right] \cdot \exp\left[-i\Delta\phi \cdot \operatorname{rect}\left(\frac{x - \lambda Z_i \mu / 2}{\Delta x}\right)\right] dx \quad (17)$$

which equals to:

$$H(\mu,0) = \begin{cases} 1 - \frac{3\lambda Z_i \mu}{2b} + \frac{\lambda Z_i \mu \cos \Delta\phi}{b} & |\mu| \leq \frac{\Delta x}{\lambda Z_i} \\ 1 - \frac{\Delta x}{b} - \frac{\lambda Z_i \mu}{2b} + \frac{\Delta x \cos \Delta\phi}{b} & \frac{\Delta x}{\lambda Z_i} < |\mu| \leq \frac{b}{\lambda Z_i} - \frac{\Delta x}{\lambda Z_i} \\ \frac{\lambda Z_i \mu}{2b} - \frac{\Delta x}{2b} + \frac{(b - \lambda Z_i \mu + \Delta x/2) \cos \Delta\phi}{b} & \frac{b}{\lambda Z_i} - \frac{\Delta x/2}{\lambda Z_i} < |\mu| \leq \frac{b}{\lambda Z_i} + \frac{\Delta x/2}{\lambda Z_i} \\ 1 - \frac{\lambda Z_i \mu}{2b} & \frac{b}{\lambda Z_i} + \frac{\Delta x/2}{\lambda Z_i} < |\mu| \leq \frac{2b}{\lambda Z_i} \end{cases} \quad (18)$$

Observing Eqs. 16 and 18 makes the optimization according to criteria of Eq. 8 very simple. One may derive the expression according to $\Delta\phi$ or just to plot them versus a range of values chosen for $\Delta\phi$ and see when the criterion of maximized minimum is fulfilled (Eq. 8). Plotting such a graph reveals that an optimum is obtained for $\Delta\phi = \pi/2$. As previously mentioned the value of Δx is chosen such that it will be much larger than the optical wavelength in order to avoid chromatic distortions and dispersion. In the performed simulations we chose Δx to be 1/8 of the lens aperture.

3. Experimental Verification

Several experiments involving various configurations were performed in order to verify our EDOF approach. All of them were conveyed under a polychromatic and spatially incoherent illumination.

In the first experiment a regular 4- F imaging system was constructed. Two lenses with focal length of 90mm were used. In order to demonstrate the EDOF effect, the distance of object to first lens was modified to obtain out of focus equivalent to $\psi=17$. The aperture of the lenses was $D=16$ mm. In the results presented in Fig. 1 we have imaged a colored object containing features as well as letters. In Fig. 1a the images were captured when the object is in focus. Left part is without the EDOF element and the right part is with it. Fig. 1b shows the results when the object is positioned in an out of focus plane in which $\psi=17$ (shift of +2mm aside from the in-focus plane).

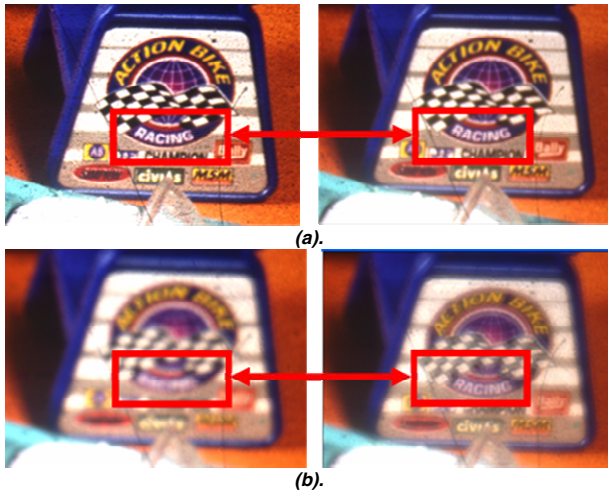
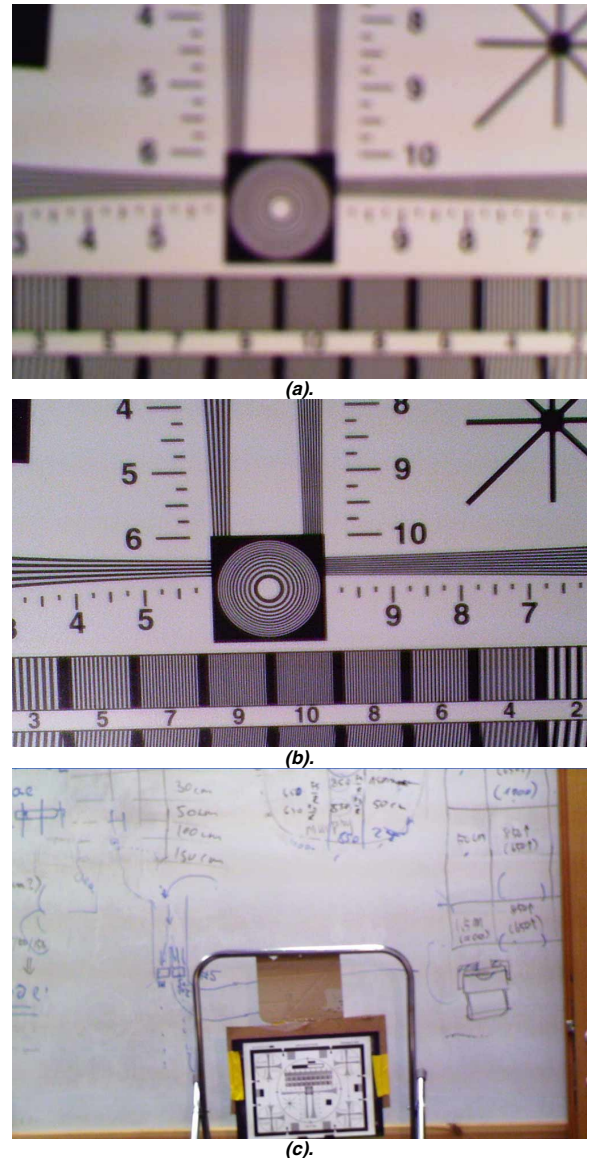


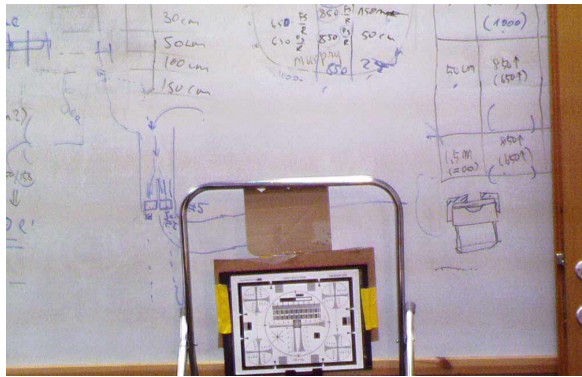
Figure 1. (a). The images were captured when the object is in focus. Left part is without the EDOF element and the right part is with it. **(b).** The same as Fig. 2a but this time the object is positioned in an out of focus plane in which $\psi=17$ (shift of +2mm aside from the in focus plane).

Note that the presented experiments were all-optical and no digital post processing was applied on the images.

In the following we used our EDOF element with an imaging module of a cellular phone camera. The EDOF element

was placed in the entrance pupil of the imaging lens. The camera has focal length of around 4.8mm and F number of 3. The device operates such that it can have a relatively focused image starting from 50cm up to infinity. By utilizing the EDOF element we tried to reduce the minimal imaging distance down to 15cm, and yet have good focused image at large distances. Fig. 2 shows the results obtained in such an all-optical experiment, where a common resolution chart was used as a target. In Fig. 2a we present the image seen when the resolution target is placed at 15cm and no EDOF element is in use. One may see that the maximal resolution obtained equals to less than 300 television lines. Fig. 2b is the result obtained when the EDOF element is inserted into the entrance pupil of the imaging module of the cellular phone camera. The object is still at 15cm. One may see that now the finest resolved feature is 800 television lines. In Figs. 2c and 2d we show the images obtained respectively without and with the EDOF element, while the resolution chart was positioned 150cm away. One may see the sharpening of the writings on the blackboard as well as the features seen in the television resolution target.





(d).

Figure 2. Close and far range all optical experiments with television resolution target. (a). Without EDOF element and object at 15cm. (b). With EDOF element and object at 15cm. (c). Without EDOF element and object at 150cm. (d). With EDOF element and object at 150cm.

We have performed another experiment with the same system of cellular camera. This time a business card was positioned 15cm away from the camera while various high resolution features were placed at a distance of 150cm. The aim of this experiment is to demonstrate that the focused images of near (the card) and far (the background shapes) fields are achieved simultaneously. Fig. 3a presents the results obtained without the EDOF element. As one may see the business card can not be read. When the EDOF element was added to the system, we obtained images showed in Fig. 3b. The business card is now readable. In order to sharpen the results we have applied sharpening and de-noising algorithms over the image. However note that the processing algorithms that we used were of very low computational complexity and included mainly summations and subtractions.

4. Conclusions

In paper we have demonstrated a novel approach providing significantly increased depth of focus i.e. axial super resolving imaging while a binary phase only element with low spatial frequency is used in the entrance pupil of the imaging lens. The proposed approach not only increases the energetic efficiency but also reduces the sensitivity to wavelengths. The element was theoretically as well as experimentally investigated. A real cellular based imaging system was constructed and tested. The results provided by the element are done in an all-optical way and thus it may fit to ophthalmic applications as well [13].



(a).



(b).

Figure 3: (a). Without EDOF element. (b). With EDOF element and after applying digital processing.

References

- [1]. W. T. Cathy and E. R Dowski, "Apparatus and method for extending depth of field in image projection system", 6069738 (May 2000).
- [2]. W. T. Cathy and E. R Dowski, "Extended depth of field optical systems", WO 99/57599 (November 1999).
- [3]. W. T. Cathy, "Extended depth field optics for human vision", WO 03/052492 (June 2003).
- [4]. E. R Dowski and W. T. Cathey, "Extended depth of field through wave-front coding" Appl. Opt 34, 1859 (1995).
- [5]. C. M. Hammond, "Apparatus and method for reducing imaging errors in imaging systems having an extended depth of field", 6097856 (August 2000).
- [6]. D. Miller and E. Blanko, "System and method for increasing the depth of focus of the human eye, 6554424 (April 2003).
- [7]. N. Atebara and D. Miller, "Masked intraocular lens and method for treating a patient with cataracts", 4955904 (September 1990).
- [8]. J. O. Castaneda, E. Tepichin and A. Diaz, "Arbitrary high focal depth with a quasi optimum real and positive transmittance apodizer", Appl. Opt. 28, 2666-2669 (1989).
- [9]. J. O. Castaneda and L. R. Berriel-Valdos, "Zone plate for arbitrary high focal depth", Appl. Opt. 29, 994-997 (1990).
- [10]. E. Ben-Eliezer, Z. Zalevsky, E. Marom, N. Konforti and D. Mendlovic, "All optical extended depth of field imaging system", WO 03/076984 (September 2003).
- [11]. E. Ben-Eliezer, Z. Zalevsky, E. Marom and N. Konforti, "All-optical extended depth of field imaging system", J. Opt. A: Pure Appl. Opt. 5, S164-S169 (2003).
- [12]. E. Ben-Eliezer, E. Marom, N. Konforti, and Z. Zalevsky, "Experimental realization of an imaging system with an extended depth of field," Appl. Opt., 44, 2792-2798 (2005)
- [13]. Z. Zalevsky, "Optical method and system for extended depth of focus," 10/97494 (August 2004)

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